

HEAVY-ELEMENT ABUNDANCES AND THE INTERPRETATION
OF GLOBULAR CLUSTER CHARACTERISTICS*

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ABSTRACT

A grid of evolutionary models has been constructed for all permutations of the composition parameters $Y = 0.35, 0.20, 0.10, 0.0$ and $Z = 10^{-3}, \frac{2}{3} \times 10^{-4}, 10^{-5}$. Using time-constant loci derived from this grid, a relationship is established between cluster age, initial composition of cluster stars, and the luminosity of stars near cluster turnoff. It is found that a variation in Z by a factor of 5 can affect the determination of cluster age by as much as a variation in Y from 0.35 to 0.10.

By comparing the absolute magnitude of stars near "turnoff" in a globular cluster with model time-constant loci, it is in principle possible to determine the age of the cluster, once the chemical composition is known. Alternately, once age and chemical composition are specified, it is in principle possible to determine the absolute magnitude of cluster stars. Finally, with age and turnoff specified, composition may in principle be determined. The major reason for focusing on turnoff magnitude is that the luminosity at the theoretical turnoff point is relatively insensitive to the particular treatment of envelope convection.

The dependence of age and turnoff point on the initial helium-to-hydrogen ratio has been discussed by Demarque (1967) and by Faulkner and Iben (1967). We wish here to point out that, for any choice of cluster turnoff luminosity, the choice of heavy-element abundance (insofar as it influences opacity and energy generation via the CN cycle) also critically affects the derived age. This result might at first seem surprising since, in low-mass Population II models near the main sequence, energy production is primarily via the p - p -chain reactions and, over much of the interior, heavy elements provide only a minor contribution to opacity. However, we find that, as turnoff luminosity is approached, the CN-cycle reactions provide a sufficiently large contribution to the energy-production rate and the influence of heavy elements on opacity near 10^6 °K is sufficiently large that a variation of the parameter Z (within Population II limits) results in a significant change in stellar radius. As evolution progresses, the distinction in the Hertzsprung-Russell diagram between two models of a given mass and initial X , but specified by different values of Z , becomes quite pronounced, in contrast to the situation near the main sequence.

We have constructed a large grid of evolutionary sequences differentiated as to composition. In each case we have taken the abundance of the CNO elements, Z_{CNO} , to be 0.6 of the Z chosen for opacity purposes. We find that within the range $Z = 10^{-5} \rightarrow 10^{-3}$, a factor of 5–10 in the assumed value of Z alters the derived age as much as a change in the initial helium abundance from $Y = 0.1$ to $Y = 0.35$. For example, if cluster turnoff

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luminosity should satisfy $\log (L_{to}/L_{\odot}) \cong 0.4$, and the initial helium abundance should be $Y = 0.35$, we find that an increase in Z from $\frac{5}{3} \times 10^{-4}$ to 10^{-3} reduces the derived age from 12×10^9 years to 9×10^9 years. Thus a knowledge of initial heavy-element abundances (as well as of the hydrogen-to-helium ratio) is essential for the determination of cluster age as a function of turnoff magnitude.

The results of evolutionary calculations for an initial hydrogen abundance $X = 0.65$ and for the two choices $Z = \frac{5}{3} \times 10^{-4}$ and $Z = 10^{-3}$ are shown in Figure 1.

Opacities have been interpolated from Cox-Stewart tables for: "Kippenhahn II" and "Weigert IV" mixtures ($X = 0.9$ and 0.0 ; $Z = 10^{-3}$) (Cox and Stewart 1965); "Demarque" mixtures III and IV ($X = 0.999$ and 0.0 ; $Z = 10^{-3}$); and "Cox-Stewart"

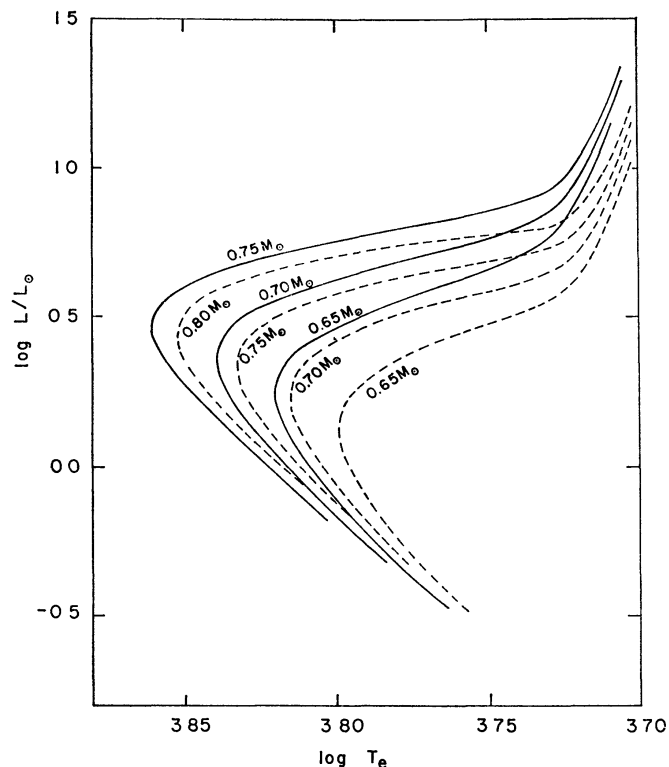


FIG. 1.—Evolutionary tracks in the Hertzsprung-Russell diagram for Population II models when initial $X = 0.65$. Solid lines are for models with $Z = \frac{5}{3} \times 10^{-4}$, and dashed lines are for models with $Z = 10^{-3}$.

mixtures I–V ($X = 0.9996, 0.8, 0.5, 0.2$, and 0.0 ; $Z = 4 \times 10^{-4}$). We are very much indebted to Drs. A. N. Cox and P. Demarque for sending us these opacity tables. Some additional opacities have been calculated for the $Z = \frac{5}{3} \times 10^{-4}$ cases, using the Cox opacity code at the Institute for Space Studies. The treatment of envelope convection which we have employed has been described by Iben (1963). The ratio of mixing length to density scale height has been fixed at $(l/H_d) = \frac{1}{2}$ for all models.

In each of the cases shown in Figure 1, the initial abundances of C^{12} , N^{14} , and O^{16} have been taken in the ratio $X_{12}:X_{14}:X_{16} = 3:1:9$. Since in each case, as the model turnoff luminosity is approached, C^{12} and O^{16} have been converted *almost completely* into N^{14} in the region of nuclear-energy production, the relative distribution within the CNO group is actually irrelevant for our purposes and Z_{CNO} remains as the only parameter that needs to be specified for the group (as far as energy-generation rates are concerned).

It will be noted from Figure 1 that, for a given initial mass, both luminosity and sur-

face temperature at model turnoff are significantly lowered by the increase in Z from $\frac{5}{3} \times 10^{-4}$ to 10^{-3} . In this particular set of cases, the track for a star of mass M_1 and $Z = 10^{-3}$ looks very much like the track for a star of smaller mass $M_2 = M_1 - 0.05 M_\odot$ and $Z = \frac{5}{3} \times 10^{-4}$. However, the *time* to reach the turnoff is primarily a function of mass and initial hydrogen abundance and is relatively insensitive to the value of Z within the Population II range. This is because, for a given mass and a given distribution of hydrogen, luminosity is (1) a function primarily only of a mean "interior" opacity (in the "interior," at temperatures above a few million degrees, the major opacity sources are Z -independent electron scattering and free-free absorption off hydrogen and helium) and (2) relatively insensitive to the precise form of the energy sources (an extension of

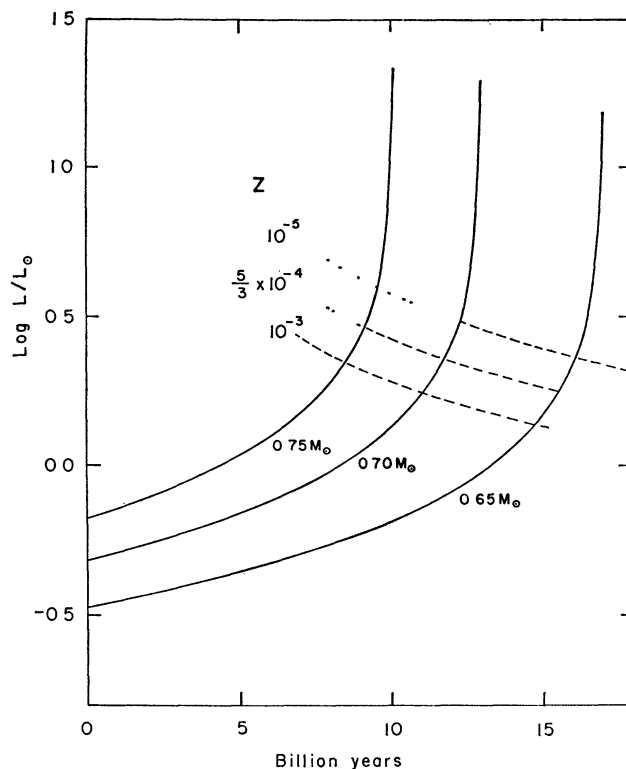


FIG. 2.—Tracks in the luminosity-age plane for Population II models when $X = 0.65$, $Z = \frac{5}{3} \times 10^{-4}$. Dashed (calculated) and dotted (extrapolated) curves give the position of model turnoff versus age at turnoff for the indicated values of Z .

the well-known result that, for main-sequence stars, the mass-luminosity relationship does not involve energy sources explicitly). Thus, for an assigned *model* turnoff luminosity, the larger Z , the larger is the appropriate model mass and the shorter is the resultant age.

In Figure 2 we exhibit model luminosity as a function of time for initial $X = 0.65$ and $Z = \frac{5}{3} \times 10^{-4}$. The dashed curve labeled $\frac{5}{3} \times 10^{-4}$ defines the locus of model turnoff points for this case. For comparison, we have also drawn loci of model turnoff points for the $Z = 10^{-3}$ ($X = 0.65$) case and for a $Z = 10^{-5}$ case, for which models have also been computed. If, for example, *model* turnoff is chosen so that $\log (L_{to}/L_\odot) = 0.4$, then the derived ages are 14.8, 10.6, and 7.5×10^9 years, respectively, for the choices $Z = 10^{-5}$, $\frac{5}{3} \times 10^{-4}$, 10^{-3} .

For comparison with clusters, information given by theoretical time-constant loci is more relevant than that given directly by individual tracks. Time-constant loci have

been prepared for the composition choices already cited as well as for additional composition sets defined by permutations of the values $X = 0.8, 0.9, 1.0$ and $Z = 10^{-3}, \frac{5}{3} \times 10^{-4}, 10^{-5}$. More details concerning models will be presented in a later communication.

The entries in Table 1 give, as a function of composition, the ages corresponding to time-constant loci with *locus* turnoff luminosity specified by $\log (L_{to}/L_{\odot}) = 0.3, 0.4$, and 0.5 . An estimate of any of the quantities necessary for the use of Table 1 is fraught with such uncertainty that we hesitate to draw any conclusions on comparison with the available observational data. Nevertheless, we offer the following examples as an illustration of the use to which the information in Table 1 might be put, were the requisite quantities known to sufficient precision.

One possibility of assigning an absolute magnitude to stars near cluster turnoff capitalizes on the fact that stars near turnoff are less luminous than RR Lyrae stars of the same color by an observationally fairly well-defined factor. Unpublished work by

TABLE 1
CLUSTER AGE IN 10^9 YEARS AS A FUNCTION OF TURNOFF
LUMINOSITY AND COMPOSITION

Z	X	LOG (L_{to}/L_{\odot})		
		0.5	0.4	0.3
10^{-3}	0.999	11.3	14.7	18.6
	0.90	9.9	12.4	16.1
	0.80	8.6	10.7	14.0
	0.65	7.0	8.8	11.4
$\frac{5}{3} \times 10^{-4}$	0.999833	14.5	18.0	22.5
	0.90	12.7	15.8	20.1
	0.80	11.3	14.0	18.0
	0.65	9.6	11.8	14.8
10^{-5}	0.99999	16.0	19.5	24.2
	0.90	15.4	18.4	22.8
	0.80	14.5	17.5	21.7
	0.65	12.6	16.0	20.2

Sandage (see Sandage and Smith 1966) indicates that, in each of the clusters M15, M92, and M13, stars near the cluster turnoff lie about 3.5 ± 0.25 mag below the RR Lyrae variables in the same cluster. Thus, if there were some way of assigning luminosities to RR Lyrae stars, the luminosity of turnoff stars could be obtained from $\log L_{to} \cong (\log L_{RR} - 1.4) \pm 0.1$.

Using the method of statistical parallax, Pavlovskaya (1953) finds a mean value $M_{pg} = 0.5 \pm 0.18$ ($M_v \sim 0.4$) for 69 RR Lyrae variables in the field. Using the convergent-point method, Eggen and Sandage (1959) obtain a mean value of $M_v = 0.6 \pm 0.1$ for five of the variables in Pavlovskaya's list. If the ordinary cosecant law of extinction can be applied to these objects, then the Eggen-Sandage result should be modified to $M_v \sim 0.3$ (Arp 1962). Selecting $M_v = 0.2-0.7$ and adopting a bolometric correction of ~ -0.1 , we have $\log (L_{RR}/L_{\odot}) \sim 1.67-1.87$. We shall choose $\log (L_{to}/L_{\odot}) \sim 0.37$.

Spectroscopic evidence suggests that the heavy-element abundance in extreme Population II stars is down from the solar value ($Z_{\odot} \sim 0.02$) by a factor of at least 40 (O'Dell, Peimbert, and Kinman 1964) and possibly by a factor of 100 or more (Helfer, Wallerstein, and Greenstein 1959; Aller and Greenstein 1960; Baschek 1962). For the sake of concrete illustration, we shall adopt $Z \sim 2 \times 10^{-4}$.

Interpolation in Table 1 (in terms of $\log Z$, X , and $\log L_{\text{to}}$) gives, with $\log (L_{\text{to}}/L_{\odot}) = 0.37$ and $Z = 2 \times 10^{-4}$, a cluster age of $t_c = 12.3, 14.7$, or 19.0 for the choices $X = 0.65, 0.8$, or ~ 1.0 , respectively. It seems reasonable to suppose that X is at least as large as that hydrogen abundance thought representative of Population I stars. If, for example, we suppose that $X \geq 0.7$ and if we adopt $Z \sim 2 \times 10^{-4}$ and $\log (L_{\text{to}}/L_{\odot}) \sim 0.37$, we must conclude that $t_c \geq 13$.

A different constraint follows naturally from the most naïve interpretation of the Hubble time. If we suppose that $t_c \lesssim 2/3t_H \sim 10$ and if we accept $Z \sim 2 \times 10^{-4}$ and $\log (L_{\text{to}}/L_{\odot}) \sim 0.37$, we conclude from Table 1 that $X \leq 0.50$. If we insist that $X \geq 0.7$, we must therefore give up one (or both) of the assertions concerning Z and $\log L_{\text{to}}$. For example, we could have either (1) $\log (L_{\text{to}}/L_{\odot}) = 0.37$, but $Z \gtrsim 1.1 \times 10^{-3}$, or (2) $Z \sim 2 \times 10^{-4}$, but $\log (L_{\text{to}}/L_{\odot}) \gtrsim 0.5$, or (3) some compromise between choices 1 and 2.

REFERENCES

- Aller, L. H., and Greenstein, J. L. 1960, *Ap J. Suppl.*, **5**, 139.
 Arp, H. C. 1962, *Ap J.*, **135**, 971.
 Baschek, B. 1962, *Zs. f. Ap.*, **56**, 207.
 Cox, A. N., and Stewart, J. N. 1965, *Ap J. Suppl.*, **11**, 22.
 Demarque, P. 1967, *Ap J.*, **149**, 117.
 Eggen, O. J., and Sandage, A. R. 1959, *M.N.R.A.S.*, **119**, 255.
 Faulkner, J., and Iben, I., Jr. 1968, *Ap J.* (in press).
 Helfer, H. L., Wallerstein, G., and Greenstein, J. L. 1959, *Ap J.*, **129**, 700.
 Iben, I., Jr. 1963, *Ap J.*, **138**, 452.
 O'Dell, C. R., Peimbert, M., and Kinman, T. D. 1964, *Ap J.*, **140**, 119.
 Pavlovskaya, E. D. 1953, *Variable Stars*, **9**, 349.
 Sandage, A. R., and Smith, L. L. 1966, *Ap J.*, **144**, 886.

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